

Algebra 2

8-01 Sample Spaces and Probability

Sample Spaces

- The set of all possible _____ in a probability experiment
 - Probability Experiment: _____
 - Sample Space: _____
 - Event (wanted outcome): _____
 - Outcome (what happened): _____

Find the number of possible outcomes and then list all the possible outcomes if you flip 4 coins.

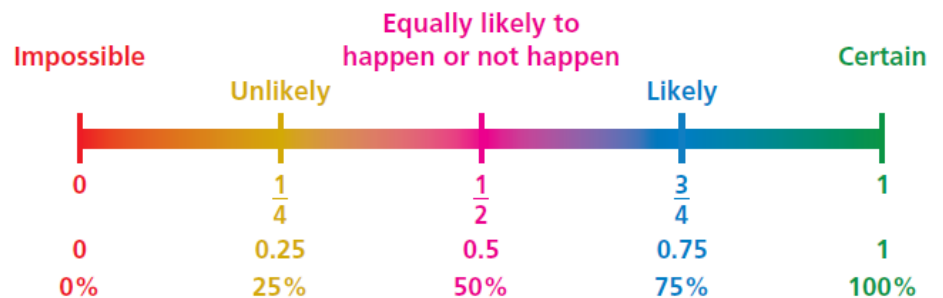


Find the number of possible outcomes and then list all the possible outcomes if you flip a coin and draw a marble at random from a bag with 2 purple marbles and 1 white marble.



Probability

- A number between _____ and _____ to indicate how likely something is to _____
- 0 = _____ happen
- 1 = _____ happens



Theoretical Probability

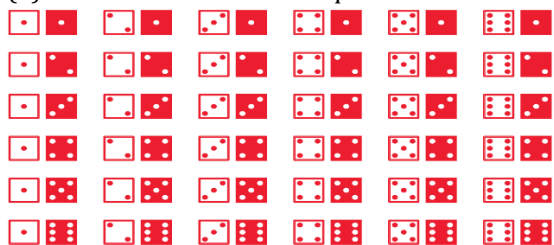
$$P(A) = \frac{\text{Number of ways A happens}}{\text{Total number of possible outcomes}}$$

You flip a coin four times. What is the probability that the coins show heads exactly three times?

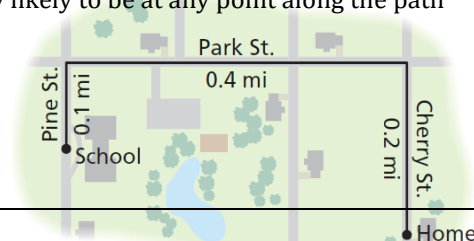
A game show airs 5 days a week. Each day a prize is randomly placed behind one of two doors. What is the probability that exactly two contestants guess the correct door during a week?

Two D6 are rolled. (a) What is the probability of rolling a sum that is *not* 2?

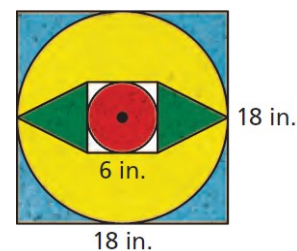
(b) The sum is less than or equal to 10?



A student loses his earbuds while walking home from school. The earbuds are equally likely to be at any point along the path shown. What is the probability that the earbuds are on Cherry Street?



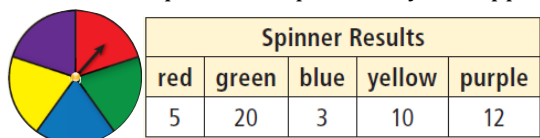
Find the probability of randomly picking a point in the yellow area.



Experimental Probability

- Probability based on the results of an _____

Each section of the spinner shown has the same area. The spinner is spun 50 times. The table shows the results. For which color is the experimental probability of stopping on the color the same as the theoretical probability?



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8-02 Two-Way Tables and Probability

Two-Way Table

- Displays data from _____ source that belongs to _____ different categories
- Entries are _____ frequencies
- Totals are _____ frequencies

		Attendance	
		Attending	Not Attending
Class	Freshman	25	44
	Sophomore	80	32

joint frequency

There are 16 juniors and 24 seniors on a debate team. Of those, 7 juniors and 19 seniors qualify for a state debate competition. Organize this information in a two-way table. Then find and interpret the marginal frequencies.

		State Competition		
		Qualified	Not Qualified	Total
Class	Jr.			
	Sr.			
	Total			

Relative Frequencies

- Joint Relative Frequency
 - Ratio of _____ frequency to _____ values
- Marginal Relative Frequency
 - Sum of _____ relative frequencies in a _____ or _____

Make a table showing the relative frequencies.

		State Competition		
		Qualified	Not Qualified	Total
Class	Jr.	7	9	16
	Sr.	19	5	24
	Total	26	14	40

		State Competition		
		Qualified	Not Qualified	Total
Class	Jr.			
	Sr.			
	Total			

Conditional Relative Frequencies

- Ratio of a _____ frequency to the _____ frequency
- Can be done for _____ totals or _____ totals

Make a two-way table that shows the conditional relative frequencies based on (a) the row totals

		State Competition		
		Qualified	Not Qualified	Total
Class	Jr.	7	9	16
	Sr.	19	5	24
	Total	26	14	40

		State Competition	
		Qualified	Not Qualified
Class	Jr.		
	Sr.		

415 #1, 2, 3, 5, 7, 9, 11, 13, 14, 15, 19, 21, 25, 27, 29 = 15

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8-03 Conditional Probability

Conditional Probability

- Probability that B occurs given that A has already occurred

$$P(B|A)$$

A family has three rabbits and two guinea pigs. They randomly select a pet to get brushed and then randomly select a different pet to get a treat. Find the probability that they select a rabbit to get a treat given that they selected the guinea pig to get brushed.

A quality-control inspector checks for defective parts. The two-way table shows the results. Find each probability.

$P(\text{pass} | \text{defective})$

		Result	
		Pass	Fail
Part Type	Defective	5	24
	Non-defective	208	9

$P(\text{pass} | \text{non-defective})$

Conditional Probability Formula

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Find $P(\text{pass} | \text{non-defective})$ using the formula for conditional probability.

		Result	
		Pass	Fail
Part Type	Defective	5	24
	Non-defective	208	9

At a clothing store, 75% of customers buy a pair of pants, 24% of customers buy a belt, and 20% of customers buy a pair of pants and a belt.

What is the probability that a customer who buys a pair of pants also buys a belt?

What is the probability that a customer who buys a belt also buys a pair of pants?

An airline company strives to not lose luggage. A manager is evaluating three flights in order to determine which flight loses luggage the least often. At the end of each day, the manager records whether or not luggage was lost on the flights that day. The table shows the results. Which flight loses luggage the least often?

Flight	Lost Luggage	No Lost Luggage
A	 	
B	 	
C	 	I

422 #1, 3, 5, 7, 9, 10, 11, 13, 15, 16, 17, 18, 19, 21, 23, 29, 31, 33, 35, 37 = 20

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8-04 Independent and Dependent Events

- _____ events \rightarrow _____ outcomes

Independent Events

- One event _____ affect the other event
- $P(A \text{ and } B) = P(A) \cdot P(B)$
- $P(A|B) = P(A)$ and $P(B|A) = P(B)$

Dependent Events

- One event _____ affect the other event
- $P(A \text{ and } B) = P(A) \cdot P(B|A)$

A bag contains six pieces of paper, numbered 1 through 6. You randomly select a piece of paper, replace it, and then randomly select another piece of paper. Use a sample space to determine whether randomly selecting a 5 first and randomly selecting an odd number second are independent events.

A bag contains six pieces of paper, numbered 1 through 6. You randomly select a piece of paper, set it aside, and then randomly select another piece of paper. Use a sample space to determine whether randomly selecting an even number first and randomly selecting a 4 second are independent events.

A store surveys customers of different ages. The survey asks whether they want to see the store expand its toy department. The results, given as joint relative frequencies, are shown in the two-way table. Determine whether wanting to see the store expand and being less than 10 years old are independent events.

		Age (in years)		
		< 10	10–20	> 20
Response	Yes	0.27	0.06	0.23
	No	0.09	0.17	0.18

Find the probability that you get an even number on your first spin and a number less than 3 on your second spin.



Nine women and six men are on a committee. One person is randomly selected to be the chairperson and a different person is randomly selected to be the treasurer. Find the probability that both events A and B will occur.

Event A : The chairperson is a man.

Event B : The treasurer is a woman.

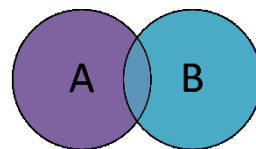
430 #1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 41 = 20

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8-05 Probability of Disjoint and Overlapping Events

Compound Event

- _____ event with _____ acceptable outcomes
- There may be some intersections where _____ condition satisfies _____ events so the events are _____
- If there is no intersection, then they are _____ or _____



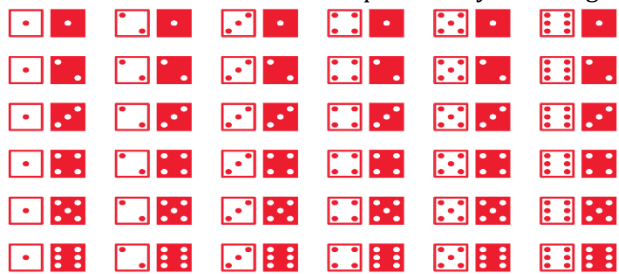
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Disjoint or mutually exclusive

$$P(A \text{ and } B) = 0$$

One D6 is rolled. What is the probability of rolling a multiple of 3 or 5?

Two D6 are rolled. What is the probability of rolling a sum that is a multiple of 2 or 3?



A bag contains twenty cards, numbered 1 through 20. A card is randomly selected. What is the probability that the number is a multiple of 3 or a multiple of 4?

Out of 45 customers at a breakfast café, 42 customers bought either coffee or orange juice. There were 30 customers who bought orange juice and 40 customers who bought coffee. What is the probability that a randomly selected customer bought both coffee and orange juice?

A medical association estimates that 10.9% of the people in the United States have a thyroid disorder. A medical lab develops a simple diagnostic test for the disorder that is 96% accurate for people who have the disorder and 99% accurate for people who do not have it. The medical lab gives the test to a randomly selected person. What is the probability that the diagnosis is correct?

437 #1-30 odd = 15

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8-06A Permutations and Combinations

Permutation

- How many ways to _____ objects

A, B, C

- Number of Permutations of n objects taken r at a time

$${}_nP_r = \frac{n!}{(n-r)!}$$

- Factorial (!) – that number _____ all whole numbers _____ than it
- $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 =$ _____

Permutations on a Calculator

TI

- Enter value of n
- Press MATH \rightarrow PRB \downarrow nPr
- Enter value of r

NumWorks

- Press Toolbox button
- Down to Probability
- Down to Combinatorics
- Permute(n,k)
- Enter n then r

Evaluate ${}_5P_2$

Evaluate ${}_9P_1$

Consider the letters in the word PENCILS.

In how many ways can you arrange all of the letters?

In how many ways can you arrange 3 of the letters?

Eight people serve on a committee. In how many different ways can a chairperson, a recorder, and a treasurer be chosen from the committee members?

You and your friend are auditioning for a part in the school play. There are 15 people auditioning, and the order of their auditions is chosen at random. Find the probability that your audition is last and your friend's audition is second to last.

You and your friend are 2 of 8 servers working a shift in a restaurant. At the beginning of the shift, the manager randomly assigns one section to each server. Find the probability that you are assigned Section 1 and your friend is assigned Section 2.

Combination

- Arranging of objects _____ order

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

- _____ have order
- _____ do not have order

Combinations on a Calculator**TI**

1. Enter value of n
2. Press MATH \rightarrow PRB \downarrow nCr
3. Enter value of r

NumWorks

1. Press Toolbox button
2. Down to Probability
3. Down to Combinatorics
4. $\binom{n}{k}$
5. Enter n then r

Evaluate ${}_5C_1$ Evaluate ${}_9C_9$

Count the possible combinations of 4 letters chosen from the list P, Q, R, S, T, U.

You are listening to music. You have time to listen to 3 songs from your playlist of 16 songs. How many combinations of 3 songs are possible?

A team of 25 rowers attends a rowing tournament. Five rowers compete at a time. How many combinations of 5 rowers are possible?

Tell whether to use a permutation or combination, then answer the question.

To complete an exam, you must answer 8 questions from a list of 10 questions. In how many ways can you complete the exam?

Fifty-two athletes are competing in a bicycle race. In how many orders can the bicyclists finish first, second, and third?

An art teacher has selected 13 projects, including one of yours and one of your friend's, to put into a display case in the hallway. The projects are placed at random. There is room for 2 projects in the middle row of the case. What is the probability that your project and your friend's project are the 2 placed in the middle row?

You and your friend are in the studio audience on a game show. From an audience of 300 people, 2 people are randomly selected as contestants. What is the probability that you and your friend are chosen?

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8-06B the Binomial Theorem

Binomial Theorem

$$(a + b)^n = {}_nC_0 a^{n-0} b^0 + {}_nC_1 a^{n-1} b^1 + \cdots + {}_nC_r a^{n-r} b^r$$
$$= \sum_{r=0}^n {}_nC_r a^{n-r} b^r$$

Expand $(c - 4)^5$

Expand $(w^3 - 3)^4$

Expand $(x + 2)^3$

Find the coefficient of the x^4 term in $(x - 3)^7$.

Find the coefficient of the x^5 term in $(x - 2)^{10}$.

445 #47, 48, 49, 51, 53, 55, 56, 57, 58, 59, 67, 71, 83, 85, 87 = 15

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8-07 Binomial Distributions

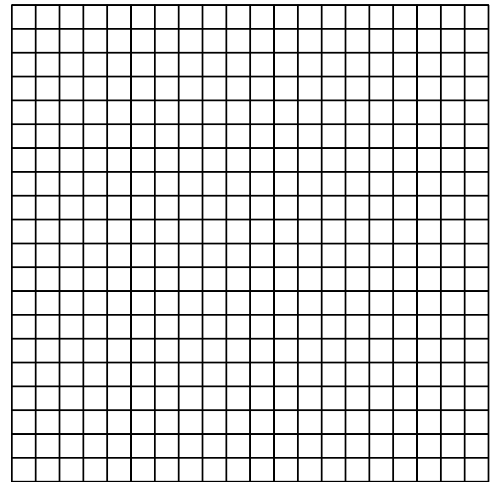
Probability Distribution

- _____ that gives the probability of _____ of the possible outcomes in a probability experiment
- The sum of the probabilities = _____

Construct a Probability Distribution

1. Make a table of _____
2. Make a _____

The spinner is divided into three equal parts. Let x be a random variable that represents the sum when the spinner is spun twice. Make a table and draw a histogram showing the probability distribution for x .



What is the most likely sum when spinning the three-section spinner twice?

What is the probability that the sum of the two spins is odd?

Binomial Distributions

- Two outcomes: _____ or _____
- Independent _____ (n)
- Probability for success is the _____ for each trial (p)

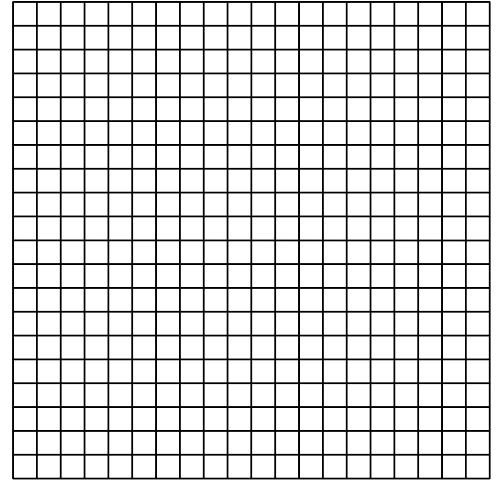
$$P(k \text{ successes}) = {}_n C_k p^k (1 - p)^{n-k}$$

Calculate the probability of flipping a coin 20 times and getting 3 heads.

only 1 head

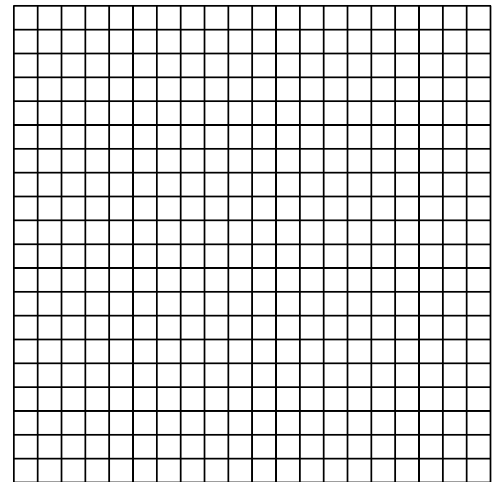
At college, 53% of students receive financial aid. In a random group of 9 students, what is the probability that exactly 5 of them receive financial aid?

Draw a histogram of binomial distribution of students and find the probability of fewer than 3 students receiving financial aid.



In your school, 30% of students plan to attend a movie night. You ask 5 randomly chosen students from your school whether they plan to attend the movie night.

a. Draw a histogram



b. Most likely

c. Probability at most 2 attend

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8-Review

Take this test as you would take a test in class. When you are finished, check your work against the answers.

8-01

1. What is the sample space for an experiment where you flip a coin and roll a dice?

8-02

2. A new flu shot was given to 73 old people and 27 young people. Of those, 15 old people and 3 young people got the flu. Organize these results in a two-way table.

Answer the following questions about this two-way table showing the results of a survey about what type of books people like to read.

	History	Literature	Total
Men	31	18	49
Women	27	24	51
Total	58	42	100

3. (a) How many women prefer to read history? (b) How many men were surveyed? (c) How many people said they preferred to read literature?
4. Rewrite the two-way table to show relative frequencies.

8-03

5. What is the probability that a person prefers to read history given that they are a woman?

8-04

Find the indicated probability

6. A and B are independent. $P(A) = 0.5$; $P(B) = 0.7$; $P(A \text{ and } B) = ?$
7. A and B are dependent. $P(A) = 0.5$; $P(A \text{ and } B) = 0.35$; $P(B | A) = ?$

8-05

Find the indicated probability.

8. $P(A) = 0.5$; $P(B) = 0.3$; $P(A \text{ or } B) = 0.7$; $P(A \text{ and } B) = ?$

8-06A

Find the number of permutations or combinations.

9. ${}_{12}P_8$
10. ${}_{12}C_8$

8-06B

Use the binomial theorem.

11. $(x + 5)^4$

8-07

Calculate the probability of k successes for a binomial experiment consisting of n trials with probability p of success on each trial. (Round to the two decimal places.)

12. $k = 6$, $n = 10$, $p = 0.8$

8-02 to 8-07

Word problems. (Round to two decimal places.)

13. If you roll one regular dice, what is the probability that you will roll a multiple of 3?

14. What is the probability that a randomly picked point in a circle with $r = 5$ in will be in a $3 \text{ in} \times 5 \text{ in}$ rectangle inside the circle?
15. If you make 80% of free throws in basketball, what is the probability you will miss a free throw?
16. Consider the numbers 1 to 10 inclusive. What is the probability that a random number is even given that it is a factor of 60?
17. If you randomly draw two cards from a bag containing 10 cards numbered 1 through 10, what is the probability of drawing a multiple of 6 and multiple of 3 without replacement?
18. If you randomly draw a single card from a bag containing 10 cards numbered 1 through 10, what is the probability of drawing a multiple of 6 or a multiple of 3?
19. What is the probability of correctly randomly guessing the answers to all 10 questions on a quiz if they are multiple choice with 5 options each?
20. If you draw 7 cards have from a bag containing 52 different cards, how many groups of 7 cards are possible?
21. If there are 20 people running for 5 class officer positions, how many different orders can there be for the class officers?
22. Three regular dice are rolled at the same time. Make a histogram showing the probability of getting each possible number of 6's. (Hint: You could get 0 6's, or 1 6, or 2 6's, or all 3 6's.)

Answers

1. H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

2.

	Flu	No Flu	Total
Old	15	58	73
Young	3	24	27
Total	18	82	100

3. 27; 49; 42

4.

	History	Literature	Total
Men	0.31	0.18	0.49
Women	0.27	0.24	0.51
Total	0.58	0.42	1

5. $P(\text{history} | \text{woman}) = 0.529$

6. 0.35

7. 0.7

8. 0.1

9. 19958400

10. 495

11. $x^4 + 20x^3 + 150x^2 + 500x + 625$

12. 0.09

13. $\frac{1}{3} \approx 0.33$

14. 0.19

15. 0.20

16. $\frac{4}{7} = 0.57$ 17. $\frac{1}{45} \approx 0.02$ 18. $\frac{3}{10} = 0.3$ 19. $\frac{1}{9765625} = 1.024 \times 10^{-7}$ (binomial distribution)

20. 133784560 (combination)

21. 1860480 (permutation)

22. $P(0 \text{ 6's}) = 0.58$; $P(1 \text{ 6's}) = 0.35$; $P(2 \text{ 6's}) = 0.07$; $P(3 \text{ 6's}) = 0.005$ 